Journal of Engineering Physics and Thermophysics, Vol. 70, No. 1, 1997

STUDY OF HEAT TRANSFER IN RECUPERATORS BY COMPUTER MODELS

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UDC 669.042.88.001.2.041:536.27

A mathematical model is developed for computer simulation of a radiative slot heat exchanger with unilateral heating. Results of mathematical simulation and experimental data are compared.

Industrial furnaces, especially high-temperature ones, are the most power-intensive elements in technological processes of machine building, metallurgy, construction, and other branches of industry. Due to high heat losses with combustion products, the efficiency does not exceed, on the average, 30% (for heating 5–15%), and the coefficient of fuel consumption is within 15–60%.

The possibility of a substantial increase in the efficiency of this type of engineering equipment by the utilization of stack-gas heat is intimately connected to the creation of reliable, compact, and universal heat exchangers (recuperators, air and water heaters).

The successful design of heat-exchanging devices is in many respects determined by the adequacy of the developed mathematical models and the specific features of heat transfer in them.

In [1-6] mathematical models of thermal processes in recuperative heat exchangers are considered, though substantial simplifications in the majority of cases reduce the value of the obtained results.

In developing a mathematical model, a unique approach to the solution of problems of heat transfer based on heat balances with allowance for possible conjugate heat transfer is used. It is assumed that the temperatures of the combustion products, air, and walls change only along the axis x. Therefore, in deriving the heat-balance equations, a heat exchanger "layer" with thickness dx at height x from its base was considered. The process was assumed to be stationary.

In the considered device, combustion products (gases) move inside a cylinder of height H and of inner and outer radii R_1 and R_2, and heated air (A) moves in a slot formed by coaxial cylinders (Fig. 1). The inner radius of the outer cylinder is R_3 and the outer radius is R_4. The outer radius of the asbestos insulation is R_5, and that of the diatomite insulation is R_6. The air-slot width is $R_3 - R_2$. The heated air absorbs heat only due to convection from the two cylinders: inner and outer. A portion of heat of the heated air is lost through the outer wall of the recuperator or insulation.

From the condition of heat balance between the heated air and heat exchanger walls within the limits of dx we obtain ($dq_a = dq_{w1} + dq_{w2}$)

$$\frac{dT_a}{dx} = \pm \frac{2\alpha_a \left[R_2 (T_{w1} - T_a) + R_3 (T_{w2} - T_a)\right]}{(R_3^2 - R_2^2)\, \nu_a c_a}, \tag{1}$$

where the "plus" sign denotes the forward flow and "minus" denotes the return flow.

Relation (1) is a differential equation for the change in air temperature over the height of the recuperator.

The gas layer loses heat due to convective heat exchange with a part of the inner wall of the heat exchanger of height dx and due to radiation through the entire wall. From the condition of heat balance we obtain an integro-differential equation for the change in gas temperature

Rostov-on-Don Institute of Automatization and Technology of Machine Building, Russia. Translated from Inzhenerno-Fizcheskii Zhurnal, Vol. 70, No. 1, pp. 117-122, January-February 1997. Original article submitted August 18, 1994.

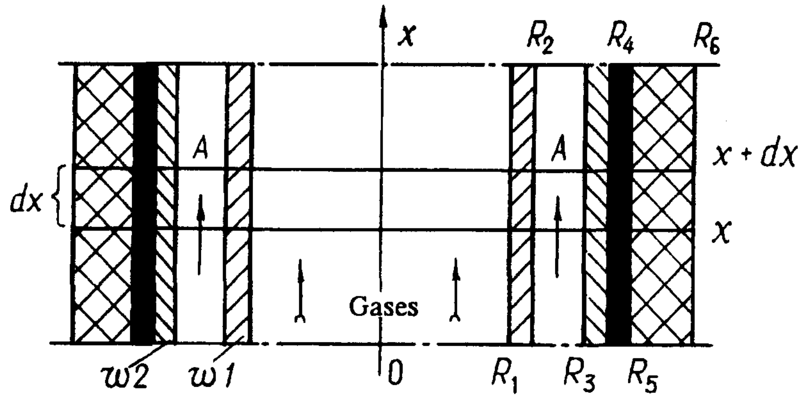


Fig. 1. On calculation of heat transfer in a radiative slot recuperator.

$$\frac{dT_g}{dx} = \frac{2}{R_1 v_g c_g}\left\{\alpha_g (T_{w1} - T_g) + \varepsilon'_w \varepsilon_g C_0 C_0 \left[\frac{1}{H}\int_0^H \left(\frac{T_{w1}}{100}\right)^4 dx - \left(\frac{T_g}{100}\right)^4\right]\right\}. \tag{2}$$

In deriving the heat-balance equations for the inner wall, we allowed for the fact that the wall takes in heat not only by convection but also by radiation, including that from the prerecuperator space (the gas duct in front of the recuperator or the furnace working area). The inner wall transfers heat to the heated air by convection and to the outer wall by radiation ($dq_{w1} = dq^c_{w1} + dq^R_{w2} + dq^{pre}_{w1} - dq^{w2}_{w1}$)

$$T_{w1} = \frac{1}{\alpha_a R_2 + \alpha_g R_1}\left\{\alpha_g R_1 T_g + \alpha_a R_2 T_a + R_1 \varepsilon'_w \varepsilon_g C_0 \left[\frac{1}{H}\int_0^H \left(\frac{T_g}{100}\right)^4 dx - \left(\frac{T_{w1}}{100}\right)^4\right] + \right.$$

$$+ \varepsilon^0_{red} C_0 \frac{2R_1^2 + x^2 - x\sqrt{4R_1 + x^2}}{2\sqrt{4R_1^2 + x^2}}\left[\left(\frac{T_{pre}}{100}\right)^4 - \left(\frac{T_{w1}}{100}\right)^4\right] +$$

$$\left. + R_2 \varepsilon^{w1}_{red} C_0 \left[\left(\frac{T_{w2}}{100}\right)^4 - \left(\frac{T_{w1}}{100}\right)^4\right]\right\}. \tag{3}$$

If the prerecuperator space is an emitting surface in the form of a circle with area πR_1^2 at level $x = 0$, we obtain with allowance for [7]

$$\varepsilon_{red} = \varepsilon^0_{red} = \frac{1}{1 + \left(\frac{1}{\varepsilon_g} - 1\right)\varphi_{g,w1} + \left(\frac{1}{\varepsilon_w} - 1\right)\varphi_{w1,g}},$$

where

$$\varphi_{w1,g} = \frac{2R_1^2 + x^2 - x\sqrt{4R_1^2 + x^2}}{2R_1\sqrt{4R_1^2 + x^2}}; \quad \varphi_{g,w1} = \frac{2R_1^2 + x^2 - x\sqrt{4R_1^2 + x^2}}{R_1^2\sqrt{4R_1^2 + x^2}}\, dx.$$

With intense radiation from the working area of the furnace, the surface temperature in front of the recuperator (at $x = 0$) cannot be specified parametrically. A formula was obtained for the heat transfer between an elementary annular surface of the inner cylinder and the furnace working area. This expression differs from the previous one as follows:

$$\varepsilon^0_{red} = \frac{\varepsilon'_{red}\,\varepsilon_{red}}{\varepsilon''_{red}\varphi_{pre,w1} + \varepsilon'_{red}},$$

where $\varepsilon'_{red} = [1/\varphi_{work,c,pre} + 1/\varepsilon_{work} - 1 + 1/\varepsilon_{pre} - 1]^{-1}$ is the reduced emissivity of the furnace working area and of the radiating surface at the recuperator base; $\varphi_{work,c,pre} = (1 + \varphi_{work,pre})/2$ is the coefficient of radiation in the presence of a reflecting (reradiating) surface and $F_{work} = F_{pre}$; ε_{work}, ε_{pre} is the emissivity of combustion products escaping from the furnace working area and in front of the heat exchanger inlet, respectively; $\varphi_{work,pre} = (2R_1^2 + h^2 - h\sqrt{h^2 + 4R_1^2})/2R_1^2$ is the angular coefficient for a round hole; h is the distance between the surfaces F_{work} and F_{pre}; $\varphi_{pre,wl} = [(2R_1^2 + x^2 - x\sqrt{4R_1^2 + x^2})/(R_1^2\sqrt{4R_1^2 + x^2})]/dx$ is the coefficient of radiation of the radiating surface (at $x = 0$) to an elementary annular surface of the inner cylinder; $\varphi'_{red} = [1 + (1/\varepsilon_{pre} - 1)\varphi'_{pre,wl} + (1/\varepsilon_w - 1)\varphi'_{wl,pre}]^{-1}$ is the reduced emissivity of the "radiating surface–inner wall of a heat exchanger" system [8]; $\varphi_{pre',wl} = H/2R_1^2(\sqrt{4R_1^2 + H} - H)$ is the coefficient of radiation of the radiating surface (circle) to the inner cylinder of the recuperator; $\varphi'_{wl,pre} = 1/4R_1(\sqrt{4R_1^2 + H^2} - H)$ is the coefficient of radiation of the recuperator wall to the radiating surface; $T_{pre} = T_{work}$ is the temperature of stack gases escaping from the furnace working area.

In deriving the heat-balance equation for the outer wall of the heat exchanger, we allowed for the fact that it receives heat from the inner wall by radiation and transfers a portion of the heat to the heated air and surrounding medium in the form of losses $(dq_{w2} = dq_{w2}^{wl} - dq_{w2}^{c} - dq_{w2}^{loss})$

$$T_{w2} = \frac{1}{R_4\alpha_{w2}^{\Sigma} + R_3\alpha_a}\left\{\alpha_a R_3 T_a + R_4\alpha_{w2}^{\Sigma}T_{sur} + R_2\varepsilon_{red}^{wl}C_0\left[\left(\frac{T_{wl}}{100}\right)^4 - \left(\frac{T_{w2}}{100}\right)^4\right]\right\}. \tag{4}$$

In the presence of insulation the temperature of the outer wall is determined by the following formula

$$T_{w2} = \frac{1}{1 + R_3\alpha_a\left[\frac{1}{\lambda_{ins}}\ln\left(\frac{R_5}{R_4}\right) + \frac{1}{\lambda_{lay}}\ln\left(\frac{R_6}{R_5}\right) + \frac{1}{\alpha_{ins}^{\Sigma}R_6}\right]} \times$$
$$\times \left\{T_{sur} + \left[\frac{1}{\lambda_{ins}}\ln\left(\frac{R_5}{R_4}\right) + \frac{1}{\lambda_{lay}}\ln\left(\frac{R_6}{R_5}\right) + \frac{1}{\alpha_{ins}^{\Sigma}R_6}\right] \times\right.$$
$$\left.\times\left[R_3\alpha_a T_a + R_2\varepsilon_{red}^{wl}C_0\left[\left(\frac{T_{wl}}{100}\right)^4 - \left(\frac{T_{w2}}{100}\right)^4\right]\right]\right\}. \tag{5}$$

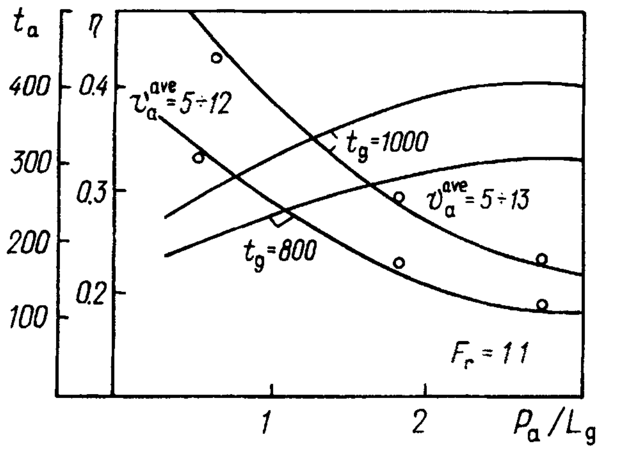
The system consisting of equations (1)-(4) or (5) is a mathematical model of a radiative slot heat exchanger with unilateral heating. To solve the system of equations, one should supplement it with the initial conditions:

$$T_a(0) = T_a^{f.b}; \quad T_g(0) = T_g^{f.b} \quad \text{– for forward flow}, \tag{6}$$

$$T_a(0) = T_a^{c}; \quad T_g(0) = T_g^{f.b} \quad \text{– for the return flow}. \tag{7}$$

The simplest (from the viewpoint of its implementation on a computer) simulation model of a radiative slot heat exchanger with unilateral heating is obtained if only convective heat transfer is taken into account. In this case, in the equations mentioned the terms allowing for gas radiation to the inner cylinder, between the walls, and from the prerecuperator space are omitted. Thus, by changing the number of terms in the mentioned equations one can determine the effect of convective and radiative heat transfer between gas and walls, of radiation between walls, and from the prerecuperator space on the process of heat transfer in the recuperator and also the choose adequate models within certain temperature ranges of combustion products.

Mathematical models of five types of a radiative slot heat exchanger were realized as a set of programs for computers of different types (Mera, ES-1033, IBM-PC (AT)) using different programming languages (FORTRAN, TURBOPASCAL, TURBOBASIC).



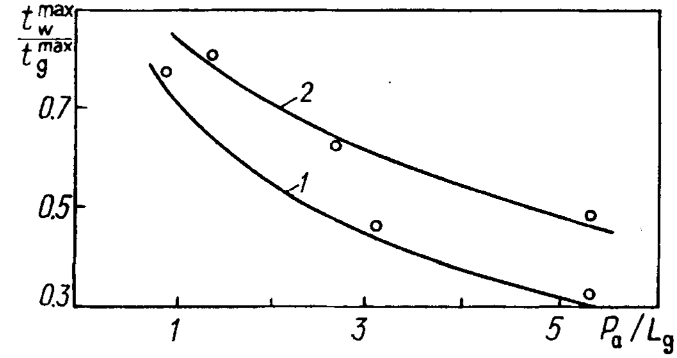


Fig. 2. Change in air-heating temperature and degree of heat recuperation as functions of relative flow rate of cooled medium: P_a and L_g, flow rate of air and combustion products, respectively, m^3/h; v_a^{ave}, real average velocity of air flow, m/sec; F_r, surface area of recuperator heating, m^2; t_g ($t = T - 273$), temperature of gases, °C. Points, experiments; curves, calculation.

Fig. 3. Relative wall temperature as a function of cooled medium flow rate (t_w^{max} and t_g^{max}, maximum temperature of inner wall and gases, respectively, °C): 1) $v_a^{ave} = 5-22$; $t_g = 800$°C; 2) 8–24 and 1200. Points, experiment; curves, calculation.

To check the results of mathematical simulation, studies were conducted on the fire bench of a radiative slot recuperator. Two series of experiments were performed: in the first the recuperator was tested without insulation of the outer cylinder; in the second, with insulation.

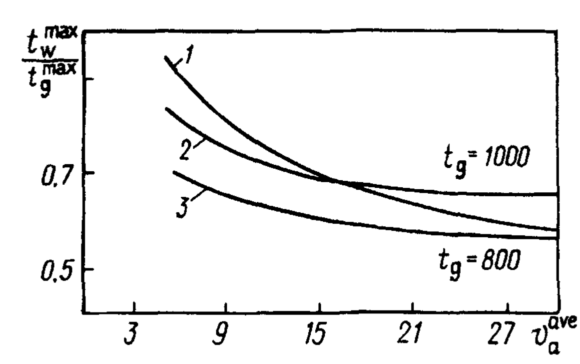
Experimental data on the temperatures of air heating, escaping gases, and recuperator walls are in good agreement with the values calculated by Eqs. (1)-(5) (the maximum discrepancy is ±7%). This discrepancy can be explained by the difference between the adopted and real schemes of heat transfer under the conditions of fire bench operation (when studying a recuperator) and also by the use of average interpolation relations in initial data of the given mathematical models (c_a, c_g, ε_w, ε_{work}, ε_{pre}).

It should be noted that the presence of integrals in the system of equations substantially complicates these equations. Numerous calculations showed that even with a very substantial change in integral mean values $1/H \int_0^H (T_g/100)^4 dx$ and $1/H \int_0^H (T_{w1}/100)^4 dx$ the temperatures of the air, gas, and walls change slightly over the entire height of the recuperator. Therefore, to simplify numerical realization the integrals in equations (2) and (3) were replaced by the values of the corresponding integrands.

Simplification of the mathematical model by reducing the number of terms in equations (1)-(5) results in an increase in the discrepancy between the experimental and calculated values of from 10 to 55%.

Comparing the results of calculations by the developed model with the data given in [1] as an example of simulation of a radiative slot recuperator, one can note that the height of the recuperator calculated by the suggested model is smaller by 340 mm than in the example, with the air heating temperature being the same. Consequently, to manufacture this recuperator one needs less (by 2.4 m^2) difficult-to-obtain high-temperature sheet steel.

Comprehensive studies on a full-scale sample and by computer model showed the following. With a ratio of the flow rates of the heated and cooled media smaller than or equal to unity, themaximum temperature of the air and recuperator-wall heating was observed, but the level of recuperation is minimal (Figs. 2 and 3). An increase in heated-medium flow rate positively affects the degree of utilization and reduction of the maximum temperature of the recuperator walls. It was found that for each value of surface heating (structural dimensions) there exists an



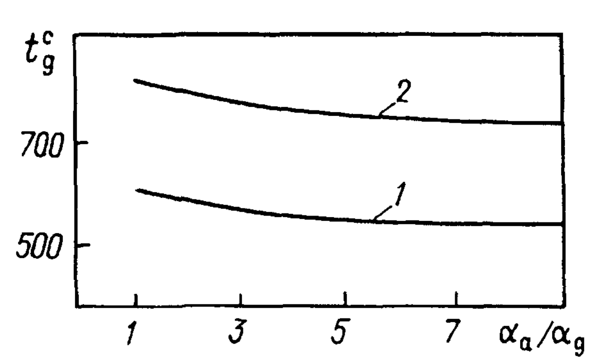


Fig. 4. Relative wall temperature as a function of real average air velocity: 1) [9]; 2, 3), calculation.

Fig. 5. Temperature of escaping gases as a function of heat transfer rate: 1) $t_g = 800°C$; 2) 1200.

optimum heated medium flow rate, which is determined by the coefficient of heat transfer and provides, the best coincidence between the required temperatures of the air and recuperator-wall heating and the efficiency of the utilizer. The calculations show that an increase in the heated surface of the recuperator by a factor of 2.3 increases the temperature of air heating by only 27–31% (depending on the temperature of the combustion products). Therefore, the increase in the degree of heat utilization by high-temperature air heating is not considered promising in recuperators of this design.

It is found by calculations (Fig. 4) that the optimum real velocity of air flow in the slot of the heat exchanger is 15 m/sec, which is in good agreement with the study of ten radiative slot recuperators with one- and two-sided heating [9].

A one-way increase in the coefficient of heat transfer from the air by increasing the flow rate and velocity of air flow has virtually no effect on the reduction of the temperature of the combustion products (Fig. 5).

CONCLUSIONS

1. Heat transfer in a recuperative device is considered with allowance for longitudinal heat transfer by radiation, the different thermophysical properties of the heated and cooled media, the change in the coefficients of heat transfer from the air and smoke over the height of the heat exchanger, the radiation from the prerecuperator space, and various schemes of heat-carrier flow.

2. On the basis of the obtained mathematical model it is possible to construct simulation models of other types of heat exchangers, including those combined with conjugate heat transfer of the elements and different surface areas of their heating.

3. The sufficient accuracy of the suggested mathematical model of thermal operation of a radiative slot heat exchanger is confirmed experimentally.

4. Simulation studies on a computer made it possible to determine ways to improve the efficiency and operation stability of a heat exchanger and also to develop engineering solutions for their practical implementation in designs of recuperators, water and air heaters, and in multipurpose heat exchangers with coordinated utilization of ther heat of combustion products of engineering devices.

NOTATION

$v_a = v_a(T_a)$, velocity of air flow, m/sec; $c_a = c_a(T_a)$, air heat capacity, J/m^3·K; T_a, air temperature, K; α_a, coefficient of heat transfer to air from inner and outer walls, W/m^2·K; T_{w1}, T_{w2}, temperature of inner and outer walls, K; T_g, T_{pre}, temperature of combustion products and prerecuperator space, K; α_g, coefficient of convective heat transfer of gases, W/m^2·K; $\varepsilon_w' = (\varepsilon_w + 1)/2$, effective emissivity of recuperator wall; $\varepsilon_g = \varepsilon_g(T_g)$, gas emissivity; C_0, constant of black body radiation, equal to 5.7 W/m^2·K^4; $\varepsilon_{red}^{wl} = [1/\varepsilon_w + (1/\varepsilon_w - 1)(R_2/R_3)]^{-1}$, reduced emissivity of system with radiation between cylinders; ε^0_{red}, generalized reduced emissivity of prerecuperator space and elementary portion of recuperator inner wall; H, heat exchanger height, m; T_{sur}, surrounding temperature, K; α^{Σ}_{w2}, total coefficient of heat transfer by radiation and convection from an outer wall to surrounding medium, $W/m^2 \cdot K$; λ_{lay}, λ_{ins}, coefficient of thermal conductivity of diatomite and asbestos of insulation, $W/m \cdot K$; α^{Σ}_{ins}, total coefficient of heat transfer of insulation surface to surrounding, $W/m \cdot K$. Indices: a, air; w, wall; g, gas; pre, prerecuperator space; red, reduced; sur, surrounding medium; lay, laying; ins, insulation; work, working; loss, losses; c, convection; R, radiation; f.b, from below; ave, averaged; r, recuperator.

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